Instanton-induced Effective Vertex in the Seiberg-Witten Theory with Matter

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ABSTRACT

The instanton-induced effective vertex is derived for $N = 2$ supersymmetric QCD (SQCD) with arbitrary mass matter hypermultiplets for the case of $SU(2)$. The leading term of the low energy effective lagrangian obtained from this vertex agrees with one-instanton effective term of the Seiberg-Witten result.

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During the past year there has been a lot of progress in $N = 2$ supersymmetric Yang-Mills theories in four dimensions. Using the idea of duality and holomorphy, Seiberg and Witten determined the exact low energy effective lagrangian for the gauge group $SU(2)$ without [1] and with matter hypermultiplets [2]. This low energy effective lagrangian is determined by a single holomorphic function: the prepotential $\mathcal{F}$. By carefully studying of its singular structures in moduli space, they determined not only the form but the numerical cefficients of the prepotential exactly.

The moduli spaces of these theories are described by hyper elliptic curves and can be related to integrable models [3]. One of the most interesting feature of this prepotential is that it contains an infinite series of instanton contributions [4]. Noticing this important feature several authors tried to check the Seiberg -Witten result with semi-classical instanton calculation at weak coupling limit without using any duality conjecture [5]. This direct microscopic instanton calculation for $N = 2$ $SU(2)$ SUSY Yang-Mills theory provides a nontrivial check of the idea of duality and has been carried out at the one-instanton [5], and two-instanton levels using the ADHM construction [6, 7].

Shortly thereafter multi-instanton calculation for SUSY Yang-Mills theory coupling to matter has been performed by the two independent groups [7, 9]. They found that there is an excellent agreement between the Seiberg-Witten result and the semi-classical instanton calculation except for the $N_f = 3, 4$ cases. At the one instanton level this microscopic calculation has been extented to the group $SU(N)$, again with and without matter [11, 12] and also to the semi-simple Lie groups [13].

Another direction of studying the instanton effects in the low energy effective lagrangian has been suggested by Yung [14]. In that approach the nonperturbative instanton effect was represented, according to the perturbation theory language, as a four fermion vertex attached to the tree level lagrangian and one can derive one instanton-induced effective vertex and find that in the low energy limit the leading term coincides with the Seiberg-Witten effective action. This provides get another nontrivial check on the exact results.

In this letter we consider the one instanton-induced effective vertex for $N = 2$ $SU(2)$

SUSY gauge theory with matter hypermultiplets which have arbitrary masses. Experience from the Seiberg-Witten theory tells us that simple addition of matter can lead to quite different structure compare to the pure Yang-Mills case [2].

The model we are considering is the $N = 2$ SQCD which has an $N = 1$ chiral multiplet $\Phi = (\phi, \psi)$ in the adjoint representation of the group $SU(2)$ and $N = 1$ vector multiplet $W_\alpha = (\lambda, v_\mu)$, which form an $N = 2$ vector multiplet. There are also $N = 1$ chiral multiplets $Q_k = (q_k, \psi_{mk})$ and $\tilde{Q}_k = (\tilde{q}_k, \tilde{\psi}_{mk})\,(k = 1, \cdots, N_f)$, which form the $N = 2$ matter hypermultiplets in the fundamental representation of the group. There exist global $SU(2)_R$ under which the superfields transform as follows:

$$\lambda \leftrightarrow \psi, \; q \to \tilde{q}^\dagger, \; \tilde{q} \to -q^\dagger, \tag{1}$$

while gauge and scalar fields are singlets under the transformation.

The lagrangian of the model is given by

$$\mathcal{L}_{SQCD} = \mathcal{L}_{SYM} + \mathcal{L}_{matter} + \mathcal{L}_{Yukawa} + \mathcal{L}_{mass}, \tag{2}$$

where each term is given as follows:

$$\mathcal{L}_{SYM} = \frac{1}{4g^2} \int d^2\theta W_\alpha^a W^{\alpha a} + \frac{1}{4g^2} \int d^2\bar{\theta} \bar{W}^{\dot{\alpha}a} \bar{W}_{\dot{\alpha}}^a, \tag{3}$$

$$\mathcal{L}_{matter} = \int d^2\theta d^2\bar{\theta} \left[\Phi^{\dagger a}(e^{-2gV}\Phi)^a + \sum_{k=1}^{N_f}(Q_k^\dagger e^{-2gV}Q_k + \tilde{Q}_k e^{2gV}\tilde{Q}_k^\dagger) \right], \tag{4}$$

$$\mathcal{L}_{Yukawa} = i\sqrt{2}g \int d^2\theta \sum_{k=1}^{N_f} \tilde{Q}_k \Phi Q_k, \tag{5}$$

$$\mathcal{L}_{mass} = \int d^2\theta \sum_{k=1}^{N_f} m_k \tilde{Q}_k Q_k. \tag{6}$$

In the above α is the Weyl index, a is the color index in the adjoint representation of the group $SU(2)$, and g is the gauge coupling constant. The first term on the RHS of eq.(4) is the adjoint representation and the second term is the fundamental representation of the matter.

The supersymmetric generalization of the instanton has been studied in the context of dynamical SUSY breaking by instanton effect with the hope that it solves the gauge

hierarchy problem [16, 17]. The defining equation of SQCD instanton in the weak coupling limit, up to the leading order of g, consists of the vector multiplets with the following equation of motion [7, 8, 9];

$$F_{\mu\nu} = -\tilde{F}_{\mu\nu}, \tag{7}$$

$$\bar{\slashed{D}}\lambda = 0, \quad \bar{\slashed{D}}\psi = 0, \quad D^2\phi - i\sqrt{2}[\lambda, \psi] = 0, \tag{8}$$

and the hypermultiplets satisfying the following equation of motion;

$$\bar{\slashed{D}}\psi_m = 0, \qquad \bar{\slashed{D}}\tilde{\psi}_m = 0, \tag{9}$$

$$D^2 q - i\sqrt{2}g\lambda\psi_m = 0, \qquad D^2\tilde{q} + i\sqrt{2}g\tilde{\psi}_m\lambda = 0, \tag{10}$$

$$D^2 q^\dagger - i\sqrt{2}g\tilde{\psi}_m\psi = 0, \qquad D^2\tilde{q}^\dagger - i\sqrt{2}g\psi\psi_m = 0. \tag{11}$$

In supersymmetric theory, due to the cancellation between bosonic and fermionic excitation modes around classical instanton background, the instanton measure takes rather a simple form that depends only on the zero mode contributions. For bosonic case there are eight zero-modes that correspond to the translation, isorotation and scale transformation. The measure takes the form [6, 14, 15, 16];

$$[d\mu_{boson}] = 2^{10}\pi^6 M^8 \rho^8 \exp\left(-\frac{8\pi^2}{g^2}\right) d^4 x_0 \left(\frac{d\rho}{\rho^5}\right) \left(\frac{d^3 u}{2\pi^2}\right), \tag{12}$$

where ρ dependence has been properly chosen to satisfy the integration measure does not have any mass dimension. There is a contribution to the measure that comes from adjoint fermion zero modes. Under the proper normalization it has the form [14]

$$[d\mu_{fermion}] = \frac{1}{16\pi^2 M} d^2\alpha \; \frac{1}{16\pi^2 M} d^2\zeta \; \frac{1}{32\pi^2 \rho^2 M} d^2\bar{\beta} \; \frac{1}{4\pi^2 \rho^2 v^2 M} d^2\bar{\eta}, \tag{13}$$

where α, ζ denote the supersymmetric mode of gaugino and higgsino, while $\bar{\beta}, \bar{\eta}$ denote the superconformal mode of the gaugino and higgsino, respectively. In addition there are another contribution comes from hypermultiplet matter zero modes [8, 9].

$$[d\mu_{hyp}] = \pi^{-2N_f} d^{N_f}\xi d^{N_f}\tilde{\xi} \equiv \pi^{-2N_f} \prod_{k=1}^{N_f} d^{N_f}\xi_k d^{N_f}\tilde{\xi}_k, \tag{14}$$

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where the Grassmann variables ξ_k and $\tilde{\xi}_k$ denotes the k-th hypermultiplet zero mode in the fundamental representation of the gauge group $SU(2)$.

To obtain full SQCD instanton measure, we should combine these three and we have

$$[d\mu_{SQCD}] = [d\mu_{boson}][d\mu_{fermion}][d\mu_{hyp}] \tag{15}$$

$$= \frac{1}{32\pi^{2+2N_f}} \frac{\Lambda^{4-N_f}}{v^2} d^4 x_0 \frac{d\rho}{\rho} \frac{d^3 u_{inv}}{2\pi^2} d^2\alpha d^2\zeta d^2\bar{\beta}_{inv} d^2\bar{\eta}_1 d^{N_f}\xi_k d^{N_f}\tilde{\xi}_k, \tag{16}$$

where the invariant orientation is defined by [14]

$$u_{inv}^{\alpha\dot{\alpha}} = u^{\alpha\dot{\beta}} \exp\left[-4i\bar{\beta}_{\dot{\beta}}\bar{\eta}^{\dot{\alpha}} - 2i\delta_{\dot{\beta}}^{\dot{\alpha}}(\bar{\eta}\bar{\beta})\right], \tag{17}$$

and SUSY invariant collective coordinates are

$$\bar{\beta}_{inv} = \bar{\beta}(1 + 4i\bar{\beta}\bar{\eta}), \tag{18}$$

$$\bar{\eta}_1 = \frac{\bar{\eta}}{1 + 4i\bar{\beta}\bar{\eta}}, \tag{19}$$

and $\Lambda = M\exp(-\frac{8\pi^2}{g^2})$ is a dynamical scale that depends on the regularization scheme. Here we use the Pauli-Villars regularization scheme, which is normally accepted in the instanton calculation. It is well known that in supersymmetric theory the β-function is only one loop effect and does not receive any higher order contribution [4] and the power change of Λ can be expected from the behaviour of one-loop β-function coeffient when the matter fields are present. To satisfy the asymptotic freedom, we restrict the value of N_f as the integer runs from 0 to 4. It can be readily checked that the measure has correct mass dimension zero.

By the way it is known that the instanton measure for $N = 1$ SUSY case can be fixed only by supersymmetry and dimensional arguments [20] and it takes the form

$$[d\mu_{N=1}]\exp(-S) = C\frac{\Lambda^4}{v^2}\exp\left(-\frac{4\pi^2}{g^2}\rho_{inv}^2 v^2\right) d^4 x_0 \frac{d\rho}{\rho}\frac{d^3 u_{inv}}{2\pi^2} d^2\theta_0 d^2\bar{\theta}_0 d^2\bar{\beta} d^2\bar{\eta}, \tag{20}$$

where C is the numerical constant that depends on the definition of the scale parameter Λ and the Grassmann variables $\theta_0, \bar{\theta}_0$ denote the supersymmetric collective coordinates and the matter fermion zero mode, repectively. Comparing eq.(20) with eq.(15) we see

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that $N = 1$ instanton measure has very similar form as that of $N = 2$. This observation leads us to use the $N = 1$ results for $N = 2$ SUSY case.

In $N = 2$ SUSY, as already mentioned, the hypermultiplet is described in the $N = 1$ chiral multiplet Q and an anti-chiral multiplet $\tilde{Q}^\dagger$, both transform as N representation of the gauge group $SU(N)$. There are another multiplets $\tilde{Q}$ and $Q^\dagger$ which transform as $\bar{N}$ representation of the group.

When the scalar matter fields have vanishing vacuum expectation values (VEV's), the matter multiplets in the one-instanton background have the following form [21, 16, 17]

$$Q_k^i = \theta^\alpha (\psi_{m\alpha})_k^i = -\theta^i \xi_k \frac{1}{\rho^2 f^{3/2}}, \qquad Q_{ik}^\dagger = \theta^\alpha (\psi_{m\alpha}^\dagger)_{ik} = -\theta_i \bar{\xi}_k \frac{1}{\rho^2 f^{3/2}}, \qquad (21)$$

and

$$Q_k^\dagger Q_k = -\theta^2 \xi_k \bar{\xi}_k \frac{1}{\rho^4 f^3}, \qquad (22)$$

where

$$f = 1 + \frac{x^2}{\rho^2}, \qquad (23)$$

where i denotes an isospin and k denotes the flavor.

For the case of the squarks have nonvanishing VEV's the product becomes [20, 21]

$$Q_k^\dagger Q_k = \frac{\tilde{x}^2 q_k^\dagger q_k}{\tilde{x}^2 + \rho^2}, \qquad (24)$$

where

$$\tilde{x}_{\alpha\dot\alpha} = (x - x_0)_{\alpha\dot\alpha} + 4i\tilde{\theta}_\alpha (\bar\theta_0)_{\dot\alpha}, \qquad (25)$$

$$\tilde{\theta}_\alpha = (\theta - \theta_0)_\alpha + (x - x_0)_{\alpha\dot\alpha} \bar\beta^{\dot\alpha}, \qquad (26)$$

and $q_k, q_k^\dagger$ are the VEV's of the k-th hypermultiplets. In fact in this case the instanton solution does not exist, but instanton-like field configuration can be obtained by the method of constrained instanton [16, 19].

In the large distance limit i.e. $x \gg \rho$, eq.(22) and eq.(24) takes the form

$$Q_k^\dagger Q_k = \begin{cases} -\theta^2 \xi_k \bar\xi_k \dfrac{\rho^2}{\tilde{x}^6} & \text{for } q_k, \; q_k^\dagger = 0, \\[4mm] q_k^\dagger q_k \left(1 - \dfrac{\rho^2}{\tilde{x}^2} \right) & \text{for } q_k, \; q_k^\dagger \neq 0. \end{cases} \qquad (27)$$

From the eq.'s (22), (24), the calculation of S_{mass} becomes [18]

$$S_{mass} = \begin{cases} -\pi^2 \sum_{k=1}^{N_f} m_k \tilde{\xi}_k \xi_k & \text{for } q_k, \, \tilde{q}_k = 0, \\ -16\pi^2 \rho^2 \sum_{k=1}^{N_f} m_k q_k \tilde{q}_k \tilde{\xi}_k \xi_k & \text{for } q_k, \, \tilde{q}_k \neq 0, \end{cases} \tag{28}$$

and it gives the additional contribution to the instanton measure.

This pattern already has been shown for $N = 1$ SUSY case [20], and also for $N = 2$ case in the Coulomb branch where the squarks have vanishing VEV's [8, 9]. These mass contributions lead to the change of prepotential that agrees with the previous result obtained by Ohta solving the Picard-Fuchs equations for $N = 2$ $SU(2)$ massive Yang-Mills theory [22].

Now let us consider the instanton-induced effective vertex, which is our main concern. Inspired by the work in Ref.[10], an instanton-induced effective vertex for $N = 2$ SUSY Yang-Mills theory has been derived, and has been shown that in the large distance limit the correlation function contracted with this vertex yields the original $N = 2$ instanton superfield [14].

Considering the correlation function of the hypermultiplet matter fields in the instanton background,

$$\langle Q_i(x_1^L, \theta_1) \, Q_j^\dagger(x_2^R, \bar{\theta}_2) \, \tilde{Q}_k^\dagger(x_3^R, \bar{\theta}_3) \, \tilde{Q}_l(x_4^L, \theta_4) \rangle, \tag{29}$$

it can also be expressed as an average over quantum fluctuations around perturbative vacuum by inserting the instanton induced effective vertex. The vertex to be added to the classical action takes the form $\exp(-V_I) = \sum_k \frac{1}{k!}(-V_I)^k$, where k-th term corresponds to the k-instanton contribution. Inserting the vertex and let us consider only the one-instanton contribution: $(-V_I^{N=2})$. When the squarks have vanishing VEV's, the product becomes

$$Q_k^\dagger Q_k = -\theta^2 \xi_k \bar{\xi}_k \psi_{mk}^\dagger \psi_{mk} + \cdots, \tag{30}$$

and when the squarks have nonvanishing VEV's, expanding $Q^\dagger$ and Q around their

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expectation values, we have

$$Q_k^\dagger Q_k = q_k^\dagger q_k + q_k^\dagger \delta Q + \delta Q^\dagger q_k + O(\delta Q^\dagger \delta Q), \tag{31}$$

where we dropped out the term quadratic in quantum fluctuations. Using the propagator for hypermultiplet matter fields,

$$\langle Q_i(x_1^L, \theta_1)\, \delta Q_j^\dagger(x_0^R, \bar{\theta})\rangle = \frac{\delta_{ij} g^2}{(2\pi)^2} e^{-2i\bar{\theta}\bar{\theta}\theta_1} \frac{1}{(x_1^L - x_0^R)^2}, \tag{32}$$

and the propagator for the Weyl fermion,

$$\langle \psi_{mi}(x_1^L, \theta_1)\, \bar{\psi}_{mj}(x_0^R, \bar{\theta})\rangle = \frac{\delta_{ij} g^2}{(2\pi)^2} e^{-2i\bar{\theta}\bar{\theta}\theta_1} \slashed{\partial} \frac{1}{(x_1^L - x_0^R)^2}, \tag{33}$$

where the left and the right coordinates are defined as

$$x_\mu^L = x_\mu + i(\bar{\theta}\bar{\sigma}_\mu\theta), \qquad x_\mu^R = x_\mu - i(\bar{\theta}\bar{\sigma}_\mu\theta), \tag{34}$$

we can deduce the form of instanton induced effective vertex.

When the squarks have the vanishing VEV's, the resulting $N = 2$ SQCD vertex in terms of $N = 1$ supermultiplets is as follows:

$$V_I^{N=2} = -\frac{1}{32\pi^{2+2N_f}} \Lambda^{4-N_f} \int d^4x_0 \frac{d\rho}{\rho} \frac{d^3 u_{inv}}{2\pi^2} \frac{1}{\Phi^{a2}} d^2\alpha d^2\zeta d^2\bar{\beta}_{inv} d^2\bar{\eta}_1 d^{N_f}\xi d^{N_f}\tilde{\xi}$$

$$\exp\left[-\frac{4\pi^2}{g^2}\rho_{inv}^2 \bar{\Phi}^a \Phi^a + \left(\frac{4\pi^2}{g^2}\right)^2 \rho_{inv}^2 \sum_{k=1}^{N_f} (Q_k^\dagger Q_k + \tilde{Q}_k^\dagger \tilde{Q}_k) - \frac{4\sqrt{2}\pi^2}{g^2}\rho_{inv}^2 \bar{\Phi}^a W_\alpha^a \zeta^\alpha \right.$$

$$+ \frac{2\pi^2}{g^2} i\rho_{inv}^2 \bar{\nabla}_{\dot{\beta}}(\bar{W}^a \bar{u}_{inv}\tau^a u_{inv})^{\dot{\beta}} - \frac{16\pi^2}{g^2}\rho_{inv}^2 (\bar{W}^a \bar{u}_{inv}\tau^a u_{inv}\bar{\beta}_{inv})$$

$$+ \frac{2\sqrt{2}\pi^2}{g^2}\rho_{inv}^2 (\zeta \slashed{D}\bar{u}_{inv}\tau^a u_{inv}\bar{\nabla}\bar{\Phi}^a) - \frac{16\sqrt{2}\pi^2}{g^2} i\rho_{inv}^2 (\zeta \slashed{D}\bar{u}_{inv}\tau^a u_{inv}\bar{\beta}_{inv})\bar{\Phi}^a$$

$$\left. + \int d^4x d^2\theta \sum_{k=1}^{N_f} m_k \tilde{Q}_k Q_k \right]. \tag{35}$$

Defining $N = 2$ chiral superfields as

$$\Psi^a(x_{N=2}^L, \theta_1, \theta_2) = \Phi^a(x_{N=2}^L, \theta_1) + \sqrt{2}\theta_{2\alpha} W^{a\alpha}(x_{N=2}^L, \theta_1) + \cdots, \tag{36}$$

$$\bar{\Psi}^a(x_{N=2}^R, \bar{\theta}_1, \bar{\theta}_2) = \bar{\Phi}^a(x_{N=2}^R, \bar{\theta}_1) + \sqrt{2}\bar{\theta}_2^{\dot{\alpha}} \bar{W}_{\dot{\alpha}}^a(x_{N=2}^R, \bar{\theta}_1) + \cdots, \tag{37}$$

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where $N = 2$ coordinates are defined as

$$(x_\mu^L)_{N=2} = x_\mu + i\bar{\theta}_1\bar{\sigma}_\mu\theta_1 + i\bar{\theta}_2\bar{\sigma}_\mu\theta_2, \tag{38}$$

$$(x_\mu^R)_{N=2} = x_\mu - i\bar{\theta}_1\bar{\sigma}_\mu\theta_1 - i\bar{\theta}_2\bar{\sigma}_\mu\theta_2. \tag{39}$$

Expanding field Ψ_a around its VEV and considering the gauge symmetry breaking

$$\Psi_a = v\delta_{a3} + \delta\Psi_a \tag{40}$$

the Ψ_1 and Ψ_2 components become massive and do not propagate at large distances [14]. Therefore the low energy theory, integrating out these heavy fields, depends only massless field Ψ_3. Then the resulting low energy effective vertex, in terms of $N = 2$ SUSY, has the form

$$
\begin{aligned}
V_I^{LE} &= -\frac{1}{4\pi^{2+2N_f}}\Lambda^{4-N_f}\int d^4x_0 \frac{d\rho}{\rho}\frac{d^3u_{inv}}{2\pi^2}\frac{1}{\Psi_3^4}d^2\theta_1 d^2\bar{\theta}_1 d^2\theta_2 d^2\bar{\theta}_2 d^{N_f}\xi d^{N_f}\tilde{\xi} \\
&\quad \exp\left[-\frac{4\pi^2}{g^2}\rho_{inv}^2\bar{\Psi}_3\Psi_3 + \left(\frac{4\pi^2}{g^2}\right)^2\rho_{inv}^2\sum_{k=1}^{N_f}(Q_k^\dagger Q_k + \tilde{Q}_k^\dagger\tilde{Q}_k)\right. \\
&\quad \left. -\frac{\pi^2}{\sqrt{2}g^2}\rho_{inv}^2 i(\bar{\nabla}_i\bar{u}_{inv}\tau^3 u_{inv}\bar{\nabla}^i)\bar{\Psi}_3 + \int d^4x d^2\theta\sum_{k=1}^{N_f}m_k\tilde{Q}_k Q_k\right],
\end{aligned} \tag{41}
$$

where new Grassmann parameters are introduced

$$\theta_1 = -\alpha, \ \ \theta_2 = \zeta, \ \ \bar{\theta}_1 = -\bar{\eta}_1, \ \ \bar{\theta}_2 = \bar{\eta}_2, \tag{42}$$

$$\bar{\beta}_{inv\dot{\alpha}} = \frac{v}{2\sqrt{2}}(\bar{u}_{inv}\tau_3 u_{inv}\bar{\eta}_2)_{\dot{\alpha}}, \tag{43}$$

and also we have used the $N = 2$ SUSY promotion rule $v \to \Psi_3$.

For the case of nonvanishing VEV's, the resulting low energy effective vertex has the form as follows:

$$
\begin{aligned}
V_I^{LE} &= -\frac{\Lambda^{4-N_f}}{64\pi^{2+2N_f}}\int d^4x_0\frac{d\rho}{\rho^{1+2N_f}}\frac{d^3u_{inv}}{2\pi^2}\frac{1}{\Psi_3^4}\prod_{k=1}^{N_f}Q_k^{-1}\tilde{Q}_k^{-1}d^2\theta_1 d^2\bar{\theta}_1 d^2\theta_2 d^2\bar{\theta}_2 d^{N_f}\xi d^{N_f}\tilde{\xi} \\
&\quad \exp\left[-\frac{4\pi^2}{g^2}\rho_{inv}^2\left(\bar{\Psi}_3\Psi_3 + \frac{1}{2}\sum_{k=1}^{N_f}(Q_k^\dagger Q_k + \tilde{Q}_k^\dagger\tilde{Q}_k)\right) - \frac{\pi^2}{\sqrt{2}g^2}\rho_{inv}^2 i(\bar{\nabla}_i\bar{u}_{inv}\tau^3 u_{inv}\bar{\nabla}^i)\bar{\Psi}_3\right. \\
&\quad \left. + \int d^4x d^2\theta\sum_{k=1}^{N_f}m_k\tilde{Q}_k Q_k\right].
\end{aligned} \tag{44}
$$

where we have used the $N = 1$ SUSY promotion rule $q_k \to Q_k$. Using the identity for any function f

$$\int d^2\bar{\theta}_1 d^2\bar{\theta}_2 f(\rho_{inv}^2) = -\frac{1}{2}\Psi_3^2 \rho^4 \left(\frac{\partial}{\partial \rho^2}\right)^2 f(\rho^2), \tag{45}$$

as for the case of without matter hypermultiplet, the integral over ρ reduced to a total derivative and there is only zero size instanton contribution [14, 20]. Carrying out the integration we have

$$V_I^{LE} = \frac{\Lambda^{4-N_f}}{16\pi^2} \prod_{i=1}^{N_f} m_i \int d^4x d^2\theta_1 d^2\theta_2 \frac{1}{\Psi_3^2}. \tag{46}$$

When we consider the matter fields Q and $\tilde{Q}$, as mentioned above, this causes the change of prepotential:

$$\mathcal{F} = -\frac{1}{8\pi^2} \prod_{i=1}^{N_f} m_i \frac{\Lambda^{4-N_f}}{\mathcal{A}^2}. \tag{47}$$

This agrees with the result obtained in Ref.[22].

To summarize the result, we have derived one instanton induced effective vertex for the case of $N = 2$ SQCD by the analogy of $N = 1$ result. The key idea used here is the behaviour of the $N = 1$ propagator in the large distance limit and its extention to the $N = 2$ SUSY case. We also used the $N = 1, 2$ SUSY promotion rule to derive the low energy effective lagrangian in the large distance limit. This promotion rule is just within the classical level and does not contain any quantum effect [14]. Under the proper normalization and the requirement of SUSY, the induced vertex takes different form that depends on the VEV's of matter field. It also depends on the representation of matter and here we concentrate on the fundamental representation.

Finally let us briefly comment the VEV's of squarks. From the previous analysis of $N = 2$ SQCD vacuum structure, it is known that for non-zero quark masses, the VEV's of squarks are zero. But when the quarks have vanishing masses and for $N_f = 0, 1$ case, the vacuum of the moduli develops only Coulomb branch but for $N_f \geq 2$, it develops the so called Higgs branches where there exist flat directions along which the gauge symmetry is completely broken [2, 23]. The explicit matrix form of these VEV's of squarks in moduli space is discussed in [23]. For further step in this direction along the

work of Novikov [21] and Ito [11, 13], the generalization of this calculation to the group $SU(N)$ or to an arbitrary Lie group has to be considered.

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